Stability of food production: EU

Stabilität der Nahrungsproduktion:EU

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Zusammenfassung

Diese Untersuchung analysiert die irreguläre Bewegung der Nahrungsproduktion in der EU. Empirische Daten der EU-Länder für die Periode 1967-2001 wurden genutzt, um das chaotische Nahrungsproduktions-Wachstumsmodell abzuschätzen. In der vorliegenden Arbeit werden folgende Ziele verfolgt: Entwicklung eines Chaos-Wachstumsmodells und Stabilitätsanalyse der Nahrungsproduktion mit Hilfe eines logistischen Wachstumsmodells in der EU für die Periode 1967-2001.
Schlagworte: Nahrungsproduktion, Wachstumsmodell, Stabilität, Chaos.

Summary

This paper analyzes an irregular movement of food production in the European Union. This analysis is oriented toward comparation of chaotic food production growth model estimated on empirical data of the European Union countries during the period 1967.-2001. The basic aims of this paper are: firstly, to set up a chaotic growth model of food production; and secondly, to analyze the stability of food production according to the presented logistic growth model in the European Union in the period 1967-2001.
Keywords: food production, growth model, stability, chaos.

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1. Introduction

Chaos theory as a set of ideas that attempts to describe structure in periodic, unpredictable dynamic systems using certain rules that can be described by mathematical equations. However, chaos theory shows the difficulty of predicting long-range behaviour of chaotic systems. Chaos theory deals with systems of equations which are able to produce motions so complex that they appear completely random. A chaotic system exhibits a sensitive dependence on initial conditions: small differences in initial conditions can generate differing outcomes.

Chaos theory started with Lorenz's (1963) discovery of complex dynamics arising from three nonlinear differential equations leading to turbulence in the weather system. Li and Yorke (1975) discovered that the simple logistic curve can exhibit very complex behaviour. Further, May (1976) described chaos in population biology. Chaos theory has been applied in Economics by Benhabib and Day (1981, 1982), Day (1982, 1983), Grandmont (1985), Goodwin (1990), Medio (1993), Lorenz (1993), among many others.

The basic purposes of this paper are: firstly, to set up a chaotic growth model of food production that is capable of generating stable equilibriums, cycles, or chaos depending on parameter values; secondly, to analyze an irregular movement of the food production growth rate in the formal framework of the logistic model, and thirdly, to analyze the stability of food production according to the presented logistic growth model in the European Union countries in the period 1967-2001.

2. A simple growth model of food production which generates chaos

Irregular movement of food production can be analyzed in the formal framework of the chaotic growth model.

The capital/output (food production) ratio, k , equals the capital stock, K, divided by output (food production), Y, or

$$k = \frac{K_t}{Y_t} = \frac{K_{t+1}}{Y_{t+1}} \qquad (1)$$

We obtain

$$k\, Y_{t+1} = K_{t+1} \qquad (2)$$

On the other hand, the marginal capital / output ratio , k_m , equals the change in capital stock, K, divided by the increase in food production, Y, or

$$k_m = \frac{K_{t+1} - K_t}{Y_{t+1} - Y_t} \qquad (3)$$

Substituting (3) into equation (2) and rearranging gives

$$k_m Y_{t+1} - k\, Y_{t+1} = k_m Y_t - K_t \qquad (4)$$

It is assumed the production function is

$$Y_t = K_t^{\,½} \qquad (5)$$

which says that aggregate output (food production) , Y_t , is an increasing function of capital ,K_t.

Substituting this into equation (4) gives

$$k_m\, Y_{t+1} - k\, Y_{t+1} = k_m\, Y_t - Y_t^{\,2} \qquad (6)$$

Solving this last equation yields the growth model

$$Y_{t+1} = \frac{k_m}{k_m - k} Y_t - \frac{1}{k_m - k} Y_t^{\,2} \qquad (7)$$

Further, it is assumed that the current value of food production is restricted by its maximal value in its time series. This premise requires a modification of the growth law. Now, the growth rate depends on the current size of food production, Y, relative to its maximal size in its time series Y^m. We introduce y as $y = Y/Y^m$. Thus y range between 0 and 1. Again we index y by t, i.e., write y_t to refer to the size at time steps t = 0,1,2,3,... Now growth rate of food production is measured by the quantity

$$y_{t+1} = \frac{k_m}{k_m - k} y_t - \frac{1}{k_m - k} y_t^{\,2} \qquad (8)$$

This model given by equation (8) is called the logistic model. For most choices of k_m, k , there is no explicit solution for (8). Namely, knowing k_m, k , and measuring y_0 would not suffice to predict y_t for any point in time, as was previously possible. It is observed that continued iteration requires higher and higher computation accuracy if we insist on exact results. In this sense, computed predictions in our model can be totally wrong.

This is at the heart of the presence of chaos in deterministic feedback processes. Lorenz (1963) discovered this effect - the lack of predictability in deterministic systems. Sensitive dependence on initial conditions is one of the central ingredients of what is called deterministic chaos.

It is possible to show that iteration process for the logistic equation

$$z_{t+1} = \pi z_t (1 - z_t) \qquad (9)$$

is equivalent to the iteration of growth model (8) when we use the identification

$$z_t = \frac{1}{k_m} y_t \quad \text{and} \quad \pi = \frac{k_m}{k_m - k} \qquad (10)$$

Using (10) and (8) we obtain

$$z_{t+1} = \frac{1}{k_m} y_{t+1} = \frac{1}{k_m} \left[\frac{k_m}{k_m - k} y_t - \frac{1}{k_m - k} y_t^{\,2} \right] =$$

$$= \frac{1}{k_m - k} y_t - \frac{1}{k_m (k_m - k)} y_t^{\,2}$$

On the other hand, using (9) and (10) we obtain

$$z_{t+1} = \pi z_t (1 - z_t) = \frac{k_m}{k_m - k} \frac{1}{k_m} y_t \left(1 - \frac{1}{k_m} y_t \right) =$$

$$= \frac{1}{k_m - k} y_t - \frac{1}{k_m (k_m - k)} y_t^{\,2}$$

Thus we have that iterating $y_{t+1} = \frac{k_m}{k_m - k} y_t - \frac{1}{k_m - k} y_t^{\,2}$ is really the same as iterating $z_{t+1} = \pi z_t (1 - z_t)$ using $z_t = \frac{1}{k_m} y_t$ and $\pi = \frac{k_m}{k_m - k}$. It is important because the dynamic properties of the logistic equation (9) have been widely analyzed (Li and Yorke (1975), May (1976)).

It is obtained that : (i) For parameter values $0 < \pi < 1$ all solutions will converge to z = 0; (ii) For $1 < \pi < 3{,}57$ there exist fixed points the number of which depends on π; (iii) For $1 < \pi < 2$ all solutions monotonically increase to $z = (\pi - 1) / \pi$; (iv) For $2 < \pi < 3$ fluctuations will converge to $z = (\pi - 1) / \pi$; (v) For $3 < \pi < 4$ all solutions will continuously fluctuate; (vi) For $3{,}57 < \pi < 4$ the solution become "chaotic" which means that there exist totally a periodic solution or periodic solutions with a very large, complicated period(e.g. see figure 1)

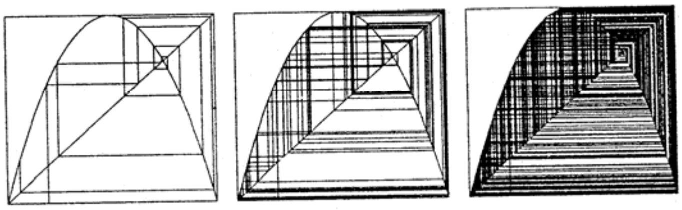

Figure 1: Iteration of the logistic map (9) for a chaotic state at $\pi = 4$
Source: PEITGEN et al. 1992, p. 59.

3. Empirical evidence

The main aim of this paper is to analyze the stability of movement of food production in the European Union countries by using the presented non-linear, logistic growth model (8).

Firstly, we transform data [FAO, 2002] on the food production, Y, from 0 to 1, according to our supposition that actual value of the food production, Y, is restricted by its highest value in the time-series, Y^m. Further, we obtain time-series of $y = Y / Y^m$. Now, we estimate the first degree auto regression models of the model (8) . (e.g. see Table 1.)

Table 1: The estimated model (8): EU

Countries	**Estimated model (9)**	**R**
European Union	y_{t+1}= 1.090326 y_t - 0.0912786 y_t^2	0.97947
Austria	y_{t+1}= 1.115416 y_t - 0.125011 y_t^2	0.93437
Belgium and Luxembourg	y_{t+1}= 1.063355 y_t - 0.060466 y_t^2	0.97915
Denmark	y_{t+1}= 1.070667 y_t - 0.070479 y_t^2	0.96086
Finland	y_{t+1}= 1.454732 y_t - 0.514956 y_t^2	0.53476
France	y_{t+1}= 1.133206 y_t - 0.142081 y_t^2	0.93537
Germany	y_{t+1}= 1.1017433 y_t - 0.1081076 y_t^2	0.94732
Greece	y_{t+1}= 1.183816 y_t - 0.198112 y_t^2	0.93582
Ireland	y_{t+1}= 1.1061356 y_t - 0.1116123 y_t^2	0.95613
Italy	y_{t+1}= 1.20996 y_t - 0.2254337 y_t^2	0.8581
Netherlands	y_{t+1}= 1.139117 y_t - 0.1483742 y_t^2	0.986
Portugal	y_{t+1}= 1.2091005 y_t - 0.2514703 y_t^2	0.77224
Spain	y_{t+1}= 1.1349637 y_t - 0.1484935 y_t^2	0.94593
Sweden	y_{t+1}= 1.416091 y_t - 0.4840604 y_t^2	0.58964
United Kingdom	y_{t+1}= 1.1193506 y_t - 0.1250157 y_t^2	0.9617

4. Conclusion

This paper has used an empirical logistic model (8) to analyze the growth stability of the food production during 1967-2001 in the European Union. This analysis of the local growth stability of food production indicates that the coefficient π and/or $\frac{k_m}{k_m - k}$ range between 1 and 2 in the estimated growth models of food production: Austria - 1.115416 (R=0.93437), Belgium and Luxembourg -1.063355 (R= 0.97915), Denmark - 1.070667 (R = 0.96086), Finland -1.454732 (

R=0.53476), France -1.133206 (R=0.93537), Germany – 1.1017433 (R= 0.94732), Greece - 1.183816 (R = 0.93582), Ireland -1.1061356 (R=0.95613), Italy -1.20996 (R=0.85181), the Netherlands - 1.139117 (R = 0.986), Portugal -1.2091005 (R=0.77224), Spain -1.1349637 (R=0.94593), Sweden - 1.416091 (R = 0.58964), UK -1.1193506 (R=0.9617), the European Union -1.090326 (R=0.97947).

This means that the estimated values of π and/or $\frac{k_m}{k_m - k}$ present the stable movement of food production toward their equilibrium values in the European Union countries in the observed period.

According to our analysis, if the index of agricultural production 1989-91 equals 100 then their equilibrium index of food production were: Austria – 102.111582 , Belgium and Luxembourg - 120.0306, Denmark – 106.282749, Finland – 94.133163, France – 101.347591, Germany – 98.0657459, Greece – 99.37151581, Ireland – 110.307991, Italy – 100.12123265, the Netherlands – 102.949479, Portugal – 88.722996, Spain – 107.1576867, Sweden – 97.820758118, UK – 96.423173965, the European Union -104.00315737.
In this sense, we can conclude that the European Union countries exibited stable movement of food production in the period 1967-2001.

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