

# ESTIMATING NON-CONCAVE METAFRONTIERS USING DATA ENVELOPE ANALYSIS

**Gunnar Breustedt, Tammo Francksen und Uwe Latacz-Lohmann**

**Department of Agricultural Economics, Christian-Albrechts University, Kiel, Germany**



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# ESTIMATING NON-CONCAVE METAFRONTIERS USING DATA ENVELOPMENT ANALYSIS

*Gunnar Breustedt\**, *Tammo Francksen\*\** und *Uwe Latacz-Lohmann\*\*\**

## Abstract

In this article we propose non-concave metafrontiers for estimating the inefficiency among production functions which do not necessarily belong to the same technology. In this case, estimating a joint production by literature approaches might be inappropriate. We call this inefficiency technological inefficiency and suggest Data Envelopment Analysis to construct a metafrontier production function which consists only of parts of different (group) frontier production functions. Thus, in contrast to the common literature our metafrontier does not need any assumptions additional to the group production functions. We illustrate our approach by means of a large sample of differently diversified crop farms. Results show that the literature approach overestimates the technological inefficiency in our sample for 75% of the observations and on average up to 7%-points in a diversification class of farms.

## Keywords

Efficiency analysis, metafrontier production function, Data Envelopment Analysis

## 1 Introduction

Optimal producer decisions must fulfil several microeconomic conditions such as optimal production level, optimal input allocation and maximizing production for a given input level to name just a few. Empirical methods to evaluate real world decisions in this context are quite common, such as efficiency analysis. However, the choice of optimal technology and related empirical research questions are not based on a common appropriate empirical approach. Analyses by Gunaratne and Leung (2000) as well as Sharma and Leung (2000), Lansink et al. (2002), Battese and Rao (2002), Battese et al. (2004) as well as Rao et al. (2004) use quite restrictive approaches. Nevertheless, questions about technology choice become more and more important in a globalized economic environment with fast evolving of new technologies. Empirical economic literature seems to concentrate on

- Estimating productivity gaps among countries and technology gaps among firms,
- Evaluating whether two technologies differ in efficiency and productivity,
- Comparing efficiencies of firms facing different technological possibilities.

Productivity and technology gaps as well as technological change in agriculture using country data have been estimated first by Hayami (1969) as well as Hayami and Ruttan (1970). Among many others, Kudaligama and Yanagida (2000) extend this approach by using efficiency analysis. It remains a matter of fact, whether one can reasonably assume the same technology for all countries as has been done in the three papers.

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\* Dr. Gunnar Breustedt, Institut für Agrarökonomie der Christian-Albrechts-Universität Kiel, Olshausenstraße 40, D-24118 Kiel – Deutschland. E-Mail: gbreustedt@agric-econ.uni-kiel.de.

\*\* Tammo Francksen, Institut für Agrarökonomie der Christian-Albrechts-Universität Kiel, Olshausenstraße 40, D-24118 Kiel – Deutschland. E-Mail: tfranck@agric-econ.uni-kiel.de.

\*\*\* Professor Dr. Uwe Latacz-Lohmann, Institut für Agrarökonomie der Christian-Albrechts-Universität Kiel, Olshausenstraße 40, D-24118 Kiel – Deutschland. E-Mail: ulatacz@agric-econ.uni-kiel.de.

In contrast, the following papers assume different technologies explicitly. Oude Lansink et al. (2002) estimate differences in technology and productivity between conventional and organic farms. Rao et al. (2004) use both the same Data Envelopment (DEA) approach like Oude Lansink et al. (2002) and the SFA (Stochastic Frontier Analysis) – the second common approach in efficiency analysis – to compute agricultural technology gaps among country groups while Battese et al. (2004) use the SFA only to explore technology gaps among garment firms that are grouped by Indonesian regions. Gunaratne and Leung (2000) as well as Sharma and Leung (2000) estimate the regional differences in technical efficiency of carp pond and shrimp production systems, respectively, by means of SFA. However, all of these papers assume (implicitly) that the different technologies can be combined.

Several other economic research questions characterized by (the optimal choice among) potentially different technologies seem to be closely related to the above examples. In Development economics or for Operations Research, one can estimate the benefits of implementing new technologies in some firms of an industry. Similarly, the costs of slow transformation of firms' institutions can be estimated, what is especially relevant in transition economies with resisting public and newly established or transformed private firms. In this context, the impact of different institutional arrangements or different objective functions of firms could be analysed, such as differences between private and public hospitals or differences between firms that are managed by the owner or by an employed manager or board. For purposes of policy analysis, the costs of (preventing from) technology switching due to subsidization, such as the payments for organic farming or renewable energies, are valuable research questions about technology comparisons, too.

In this paper, we aim at developing an empirical approach for technology comparisons that does not rely on the assumption of combinable technologies to deal with the aforementioned research problems. The remainder of the paper starts with a definition of "technological efficiency", its integration into the literature and a theoretical framework. Afterwards we describe the application of empirical methods for common efficiency analysis to estimate "technological efficiency" in the previous literature and explain a more appropriate approach. The following empirical illustration is dedicated to the problem of optimal degree of specialisation for arable farms and compares the former literature approach and the new approach. Conclusions for further applications of the concept finish the paper.

## **2 Definition of "Technological Efficiency"**

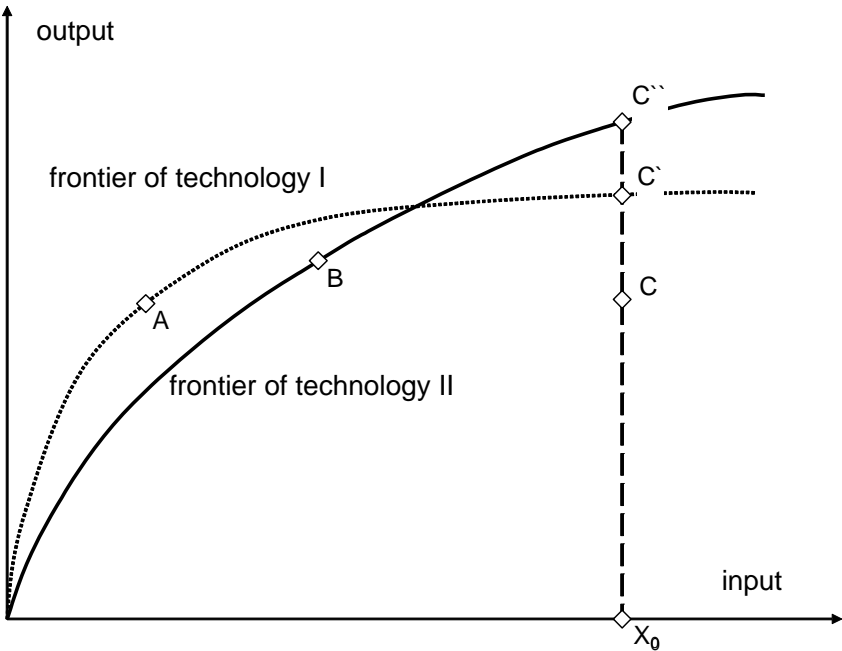
In this section we introduce the term "technological efficiency" and define related terms, relate and integrate the definitions into the literature, and finish with a theoretical framework about "technological efficiency".

From a real-world perspective, we deal with the optimal choice among different technologies and we want to estimate the productivity differences among the technologies. Although we use the term firm instead of the broader term decision unit in the following our approach is also applicable for individuals, non-profit organisations, countries and so on. We define *a firm to be "technological efficient" if it uses a technology out of the set of applicable technologies that allows for the highest possible output with a given input combination*. Or in terms of efficiency analysis, the firm uses the technology with the highest frontier output for the given inputs. In Graph 1 three firms *A*, *B*, *C* can choose between two technologies, I and II, represented by its production frontiers. Firms *A*, (*B*) and *C* use technology I (II). Only *A* is technological efficient because the frontier output of *B*'s and *C*'s actual technology is lower than the frontier output of the technology not chosen by *B* and *C*, respectively.

Similar to Gunaratne and Leung (2000), Sharma and Leung (2000), Battese and Rao (2002), Battese et al. (2004), and Rao et al. (2004) we call the frontier with the highest output for a

given input combination the metafrontier production function. “Technological inefficiency” shows a potential to increase productivity if a firm changes to the technology that determines the metafrontier for the firm’s input combination. We define “technological efficiency” as the (relative) distance between the frontier chosen and the metafrontier (= best frontier available) relative to the output of the frontier chosen. Thus, the technological efficiency of  $C$  is  $TLE_C = C^*X_0/C^**X_0$  in Graph 1 and the technological inefficiency of  $C$  is defined as  $TLI_C = 1 - TLE_C$ . Analogously, technical efficiency is the relative distance between the actual output and the frontier output for the actual technology.<sup>1</sup> We define the technical efficiency of firm  $C$   $TE_C = CX_0/C^*X_0$ .  $A$  and  $B$  are technically efficient because they are on the frontier of their technology chosen.

**Graph 1. Technical and technological efficiency**



In fact, the ratio  $TLE$  is not new. It is called “technology gap ratio” in Battese and Rao (2002) as well as in Battese et al. (2004) while O’Donnell et al. (2007) call it “metatechnology ratio”. We suggest the term “technological efficiency” analogously to “technical efficiency” because “technology gap” is more appropriate for the ratio  $TLI$  “technological inefficiency” since the technology gap should increase with the level of technological inefficiency.

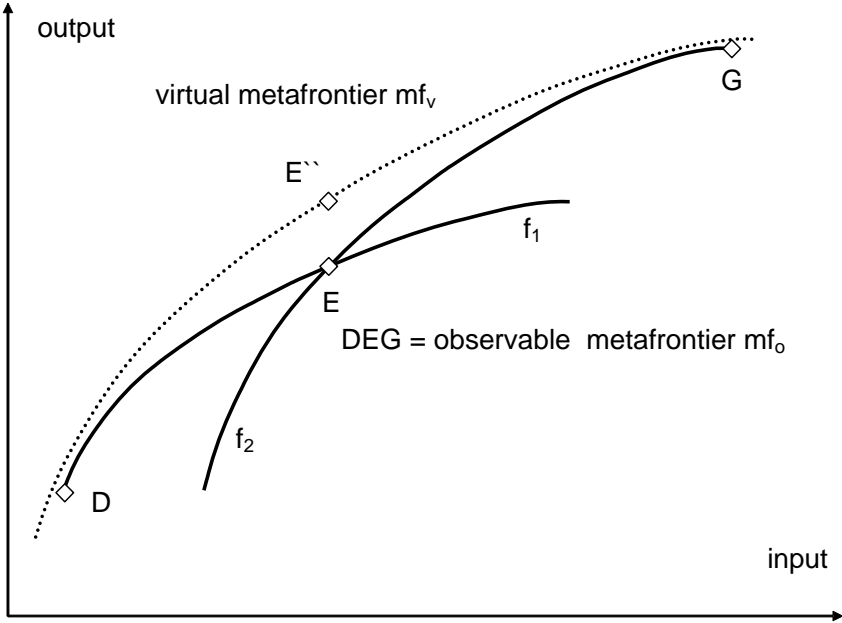
For the definition of technological efficiency the term metaproduction function needs some clarification, too. Metafrontier stands for the frontier of the metaproduction function. The term metaproduction function seems to go back to Hayami and Ruttan (1970). Following them (1971, p. 82) “the metaproduction function can be regarded as the envelopment of commonly conceived neoclassical production functions”. They define it more general as “the envelopment of all known and potentially discoverable activities” (1970, p. 898). However, if the envelopment is not applicable for all firms or countries the estimated productivity or technology gap cannot necessarily be filled because the reference given by the metaproduction function may not be reached technically. Some examples may illustrate the problem. First, there are some real-world technological problems, such as growing bananas in

<sup>1</sup> Technical inefficiency can be measured also „input-oriented“ by the relative input reduction necessary to reach the frontier in horizontal direction.

Finland or rice in the Sahara or programming computers by illiterates. Second, to estimate the metaproduction function from observable data we need some kind of aggregation, pooling, or averaging of technologies and / or observations. However, comparing, for example, farms using organic or conventional agronomic technology cannot be done by means of “pooling” or “averaging” because both technologies are mutually exclusive. Similarly, pooling, averaging or aggregating technologies in different regions may result in a technology not applicable for firms in observed regions but only for firms in some virtual “average” regions.

Hayami and Ruttan (1971) (HR) aggregate over the single technologies as can be seen from Graph 2 based on their figure 4-8 (p. 83). While HR refer to the (average) production function for each technology, we exclude the effect of technical inefficiency of firms in each technology by using the frontier production functions like in Gunaratne and Leung (2000) as well as Sharma and Leung (2000), Lansink et al. (2002), Battese and Rao (2002), Battese et al. (2004) as well as O’Donnell et al. (2007). From the single production frontiers (i.e. activities)  $f_1$  and  $f_2$ , HR construct “the envelopment of all ... discoverable activities” (1971, p. 898) like the metaproduction function  $mf_v$  which itself is not necessarily discoverable. Consequently, the outputs on  $mf_v$  may be not technically feasible and the vertical difference between  $E$  and  $E''$  is not an appropriate measure for the potential increase if the technology for  $E$  is changed. Obviously, we do not have any information in Graph 2 that the output of  $E$  can be increased by technology change since  $E$  is on the frontier of both technologies observed.  $E''$  is neither observed nor estimated by common empirical methods and, therefore, may be not reached actually. It is only a virtual reference like most other points on  $mf_v$  which we call the “virtual” metafrontier.

**Graph 2. The virtual metafrontier  $mf_v$  and the observable metafrontier DEG**



However, Gunaratne and Leung (2000) as well as Sharma and Leung (2000), Lansink et al. (2002), Battese and Rao (2002), Battese et al. (2004) as well as O’Donnell et al. (2007) construct the metafrontier such that this problem arises. The first four papers as well as O’Donnell et al. (2007) pool their observations to construct the metafrontier by means of DEA or SFA while the latter two fit completely virtual metafrontiers by enveloping the SFA group frontiers with a function of minimum sum of quadratic or absolute distances to the group frontiers. In the case of DEA the frontiers become piecewise linear and the virtual

metafrontier can consist of group frontier segments partially. However, linear combinations of several firms with different technologies can be segments of the metafrontier, too. They can be virtual because the combinations of different technologies might be technically not feasible. Thus, our definition of a technologically efficient firm differs technically from the former literature only in the set of “*applicable technologies*”. Our approach is less restrictive than the literature since we do not include the combinations of observed technologies into the set because it is hard to explore in some cases whether all technologies are combinable.

To overcome the problem of virtual reference technologies we suggest using the function represented by the graph DEG as the observable metafrontier function  $mf_o$ . At the costs of loosing a smooth function this metafrontier consists only of reference points that are technically feasible under the common assumptions of production economics and econometric methods. We now define our metafrontier  $mf_o$  following O’Donnell et al. (2007) who are closest with their definition of a virtual metafrontier to our definition of an observable metafrontier. We assume different technologies for each of  $K$  groups of firms. Each group’s distinct technology set  $T^k$  consists of all possible input-output combinations for the firms in the  $k$ -th group.

$$(1) \quad T^k = \{(x, y) : x \geq 0; y \geq 0; x \text{ can be used by firms in group } k \text{ to produce } y\}$$

whereas  $x$  ( $y$ ) is a vector of nonnegative inputs (outputs) and  $k = 1, 2, \dots, K$ . The output set  $P^k(x)$  for a given vector  $x$  using technology  $k$  is given by

$$(2) \quad P^k(x) = \{y : (x, y) \in T^k\}.$$

The boundary of this output set is called group frontier or frontier of technology  $k$ . For each group’s technology we assume some common properties of production technologies following Färe and Primont (1995):

1.  $0 \in P^k(x)$  representing the possibility of not producing;
2. If  $(x, y) \in T^k$  then  $(\theta x, y) \in T^k$  for  $\theta > 1$ , representing the possibility of wasting inputs (weak disposability);
3.  $P^k(x)$  is a closed and bounded set; and
4.  $P^k(x)$  is a convex set.

The technical efficiency relative to the group frontier  $TE^k$  is defined as the output distance function  $D^k(x, y)$

$$(3) \quad TE^k = D^k(x, y) = \inf_{\theta} \left\{ \theta > 0 : \left( \frac{y}{\theta} \right) \in P^k \right\}.$$

For a given input, this function gives the fraction of the observed output relative to the maximum output using technology  $k$ . For the metatechnology  $T^* = \{T^1 \cup T^2 \cup \dots \cup T^K\}$  follow the same characteristics except the convex output set. Thus, O’Donnell et al. (2007) must assume that the metatechnology’s output set is also convex. However, a convex metafrontier output set does not necessarily imply convex group output sets and vice versa. Therefore, we assume the metaproduction output set to be

$$(4) \quad P(x) = \{y : (x, y) \in T^*\}$$

The boundary of this output set which differs considerably from O’Donnell et al. (2007) definition is called the metafrontier  $mf_o$ . The output set that is bounded by the observed metafrontier is not necessarily a convex set. The technological efficiency of  $E$  in Graph 2 amounts to unity in our approach while it amounts to somewhat 0.8 in Rao’s et al. approach

because  $E^*$  belongs to their metafrontier. In general, the same is true for Gunaratne and Leung (2000) as well as Sharma and Leung (2000), Lansink et al. (2002), Battese and Rao (2002), Battese et al. (2004).

The advantage of our approach is not only to avoid a virtual metafrontier that might be an inappropriate reference technology but also to give a concrete recommendation which observed technology is optimal for a farm instead of a technology that might be feasible or not but that cannot be observed in reality.

### 3 Methods for Empirical Implementation

For estimating the technological efficiency we have to estimate the metafrontiers following Graph 2. The most common methods for such frontier analysis – as already mentioned above – are the Stochastic Frontier Analysis (SFA) and the Data Envelopment Analysis (DEA) to analyse the efficiency of firms. We apply DEA to construct, first, a piecewise linear frontier for each technology and, second, compute the technological efficiency based on the virtual and the observed metafrontier, respectively. The DEA approach goes back to Charnes, Cooper, and Rhodes (1978) and constructs by means of linear programming a non-parametric frontier for the analysed firms. The frontier is build up by the observations who produce the highest input for a given input combination and the convex linear combinations of the neighbouring observations with such maximum outputs.

We first present a common (i.e. for a homogenous technology) output-oriented<sup>2</sup> DEA assuming variable returns-to-scale to compute the technical efficiency. Each firm  $j$  out of  $N$  firms can produce  $s$  outputs with  $m$  inputs.

$$(5) \quad \underset{\phi_j, \lambda}{\text{Max}} \quad \phi_j \quad \text{s.t.}$$

$$\mathbf{Y}\lambda \geq \phi_j \mathbf{y}_j$$

$$\mathbf{X}\lambda \leq \mathbf{x}_j$$

$$\mathbf{e}'\lambda = 1$$

$$\lambda \geq 0$$

For each firm  $j$ , the DEA aims at maximizing a scalar  $\phi_j \geq 1$  which is multiplied by the observed output  $\mathbf{y}_j$  ( $s \times 1$  vector) to represent the maximum output that is feasible for firm  $j$ . Thus, the higher  $\phi_j$  the less efficient is firm  $j$ . The maximum output is represented by a convex linear combination of the observed outputs of all other farms  $\mathbf{Y}\lambda$  ( $s \times N$  matrix) where  $\lambda$  is a  $N \times 1$  vector of non-negative weights that have to sum to one (assumption of a variable returns-to-scale technology) and  $\mathbf{e}$  is a  $N \times 1$  vector of ones. The maximum output has to be produced with not more input than observed for firm  $j$ , i.e. the inputs corresponding to the maximum output are represented by a convex linear combination of inputs  $\mathbf{X}\lambda$  ( $m \times N$  matrix) of all firms that must not exceed the observed input  $\mathbf{x}_j$  of firm  $j$ . The technical efficiency of firm  $j$  becomes  $TE_j = \phi_j^{-1}$ .

We extend (5) to compute the technological efficiency. Thus, we have to account for several ( $K$ ) technologies one of which is termed  $k$ . For clarity, we term firm's  $j$  observed technology

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<sup>2</sup> Output oriented means that the efficiency is measured as the potential to increase the output with the same input combination. In contrast, input oriented means that the efficiency is measured as the potential to decrease the level of inputs without decreasing the output level.

$p$ , i.e.  $\mathbf{x}_j^p = \mathbf{x}_j$ ,  $\mathbf{y}_j^p = \mathbf{y}_j$ , and  $\phi_j^p = \phi_j$ . We apply two steps to solve (6) for firm  $j$  and to determine the technological efficiency.

$$(6) \quad \underset{\phi, \lambda, k}{\text{Max}} \quad \phi_j^k = \max_k \{ \phi_j^k \} \quad \text{for all } k = 1, 2, \dots, K$$

s.t.

$$\mathbf{Y}^k \boldsymbol{\lambda}^k \geq \phi_j^k \mathbf{y}_j^p$$

$$\mathbf{X}^k \boldsymbol{\lambda}^k \leq \mathbf{x}_j^p$$

$$\mathbf{e}' \boldsymbol{\lambda}^k = 1$$

$$\boldsymbol{\lambda}^k \geq 0$$

We first compute each of the distinct technical efficiencies  $\phi_j^k$  for firm  $j$ . Thus, (5) is computed  $K$  times.  $\mathbf{Y}^k$  ( $\mathbf{X}^k$ ) are the observed outputs (inputs) of all firms (except firm  $j$ ) using technology  $k$ , while the observed input and output set of firm  $j$  stay the same in each of the  $K$  computations. We receive  $\{ \phi_j^k \}$  with  $k = 1, 2, \dots, K$  and determine the maximum of  $\{ \phi_j^k \}$ . The technology which represents the metafrontier for firm  $j$  is  $T^{k^*}$  such that  $k^*$  maximizes  $\{ \phi_j^k \}$  because  $\phi_j^{k^*} \mathbf{y}_j$  represents the maximum frontier output (among the different technologies) achievable for  $j$ .

The technological efficiency of firm  $j$  becomes  $TLE_j = \phi_j^p / \phi_j^{k^*}$ . To compare the technical and the technological efficiencies we aim at another measure. Instead of  $TLE_j$  we compute  $\phi_j^{k^*} - \phi_j^p$  which represents the (relative) output increase possible by changing the technology, i.e. the distance between the frontiers of technology  $k^*$  and technology  $p$ , relative to the observed output.

The linear combinations of observations to construct a group frontier need the assumption of a piecewise linear frontier. This assumption can be criticized analogously to our critique of the convexity assumption for the metafrontier. While we fix the convexity assumption for the metafrontier by restricting it to the observable or measurable parts of the group frontiers the concept of “free disposal hull” is suggested for the DEA to relax the convexity assumption of technology. It results in a staircase frontier instead of a convex one (see Tulkens (1993) and cited literature there for details).

## 4 Empirical Illustration

### 4.1 Data

We refer to the problem of optimal specialization in the empirical illustration to compare whether our non-concave metafrontier gives other results than the virtual metafrontier approach. Thus, it is not necessary that the “technologies” of differently specialized farms are definitely not combinable. The comparison is based on data from crop farms for seven different harvesting periods (1996 – 2002). For each year separately, a farm is defined as a crop farm if output from cropping in monetary terms is more than one half of the total output of the farm without public payments. Since we look at the optimal level of specialization the observed farms are classified to five different specialization classes (i.e. technologies) according to their portion of crop output in relation to the total output. Farms are in specialization class 5 if the share of the crop output is above 50% but below 70% of the total output (Table 1). For class 4 (3) the lower bound is more than 70% (90%) and the upper



bound is 90% (less than 100%). Cropping only farms belong to class 1 (2) if they grow three or less (four and more) different crops.

To ensure homogenous farms in the sense that changing among specialization groups, i.e. technologies, is actually feasible crop farms are excluded from the analysis that show significant shares of output from lines of production with high sunk costs, such as stable for livestock. We include either crop farms which do business in cropping only or which yield revenues from agricultural service work for other farmers or non-farmers amounting to more than one half of the non-crop output. Changing the relative importance of the crop business and the service business is quite easy because the same machinery and know-how can be used.

**Table 1. Definition of specialisation classes**

specialization class (sc)	sc 1	sc 2	sc 3	sc 4	sc 5
$x = \frac{\text{monetary yield of crop production}}{\text{Gross profits of agricultural production}}$	x = 100% & more than three market fruits	x = 100% & three or more market fruits	100% > x > 90%	90% ? x > 70%	70% ? x > 50%

The total number of crop farms yearly observed in all five specialization classes ranges from a total of 303 farms in the first year to 409 farms in the last year (Table 2). The share of farms is lowest for class 1 with 4.4% on (weighted) average over the years and highest for the farms between less than 100% and more than 90% crop output. The small number of observations within a group do not cause problems in the DEA analysis since we do not aim at the technical efficiency of a farm within its group, i.e. a measure that can only decrease with the number of observations for an individual farm, but on the distance between the frontiers of different groups.

**Table 2. Shares of farms in specialisation classes (%)**

Year	sc 1	sc 2	sc 3	sc 4	sc 5	Observations
95/96	6.6	6.9	52.8	27.4	6.3	303
96/97	4.1	11.7	53.9	23.6	6.7	343
97/98	3.6	10.9	53.3	23.9	8.2	330
98/99	2.3	14.0	49.6	24.5	9.6	343
99/00	6.1	12.6	45.0	29.3	7.0	358
00/01	4.6	13.2	49.0	24.4	8.9	394
01/02	3.9	12.0	45.2	29.8	9.0	409
Average	4.4	11.7	49.6	26.2	8.0	/

The total output of a farm encloses the sales of agricultural products and agricultural services as well as the internal consumption of feed, the own consumption, the stock and inventory changes as well as subsidy payments.<sup>3</sup> The inputs are labor (measured in full time workers per year), capital (depreciations on machinery and buildings in monetary terms), costs for materials and services. Land is measured in hectare of arable and pasture land, the share of arable land is also included.

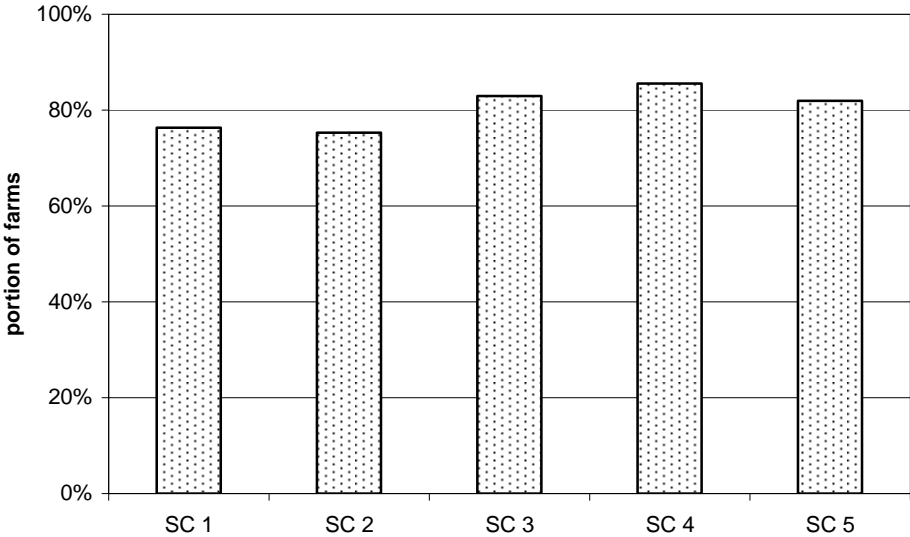
<sup>3</sup> A multi-output DEA approach is not helpful because we can aggregate the different outputs easily since they are measured in monetary terms.

The example is useful for the comparison of the virtual and the observed metafrontier approach because the five specialisation classes are exhaustive in the sense that they cover the complete range of diversification measured by the share of revenues of cropping compared to total revenues above 50%. Although, the classes are arbitrary and, thus, sections on the virtual metafrontier combined by observed farms of different specialisation classes maybe technically feasible, the virtual metafrontier approach seems to be less appropriate because of two reasons. First, although we cannot exclude feasible combinations of farms in neighbouring specialisation classes we cannot know whether such a combination is actually feasible. Second, metafrontier sections defined by farms of very different levels of specialisation are quite virtual, because a farmer cannot choose a high and low specialisation simultaneously. Note that a combination of farms with high and low specialisation does not result in a virtual farm with a medium specialisation in the DEA approach.

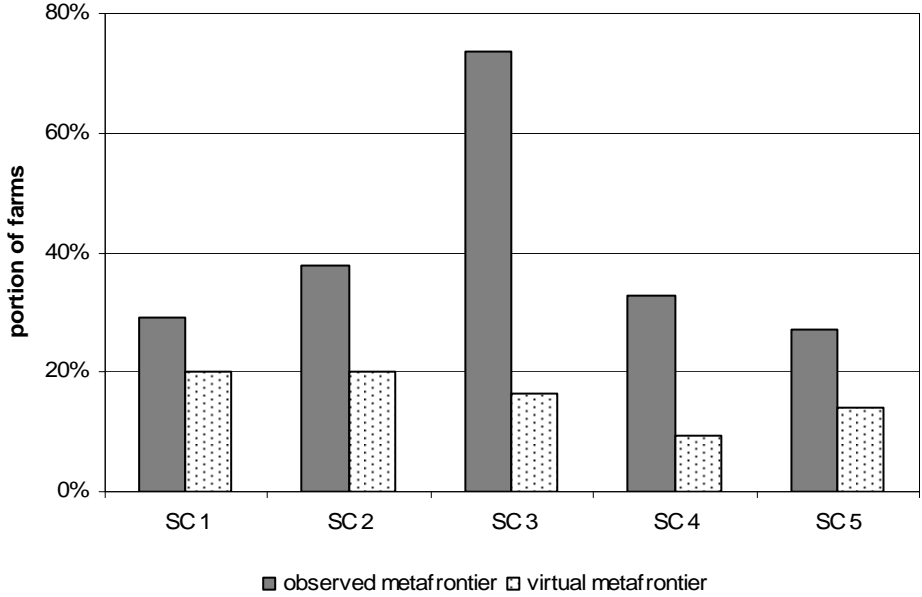
**4.2 Results and comparison with convex metafrontier**

To evaluate both metafrontier approaches, we first show the portion of farms which have a virtual metafrontier technology in Graph 3. Afterwards we compare the portions of technologically efficient farms by means of the virtual metafrontier concept and the observed metafrontier concept, respectively (Graph 4). We finally compare the technological efficiency of farms by using both approaches (Graph 5).

**Graph 3. Portion of farms with virtual metafrontier technologies**



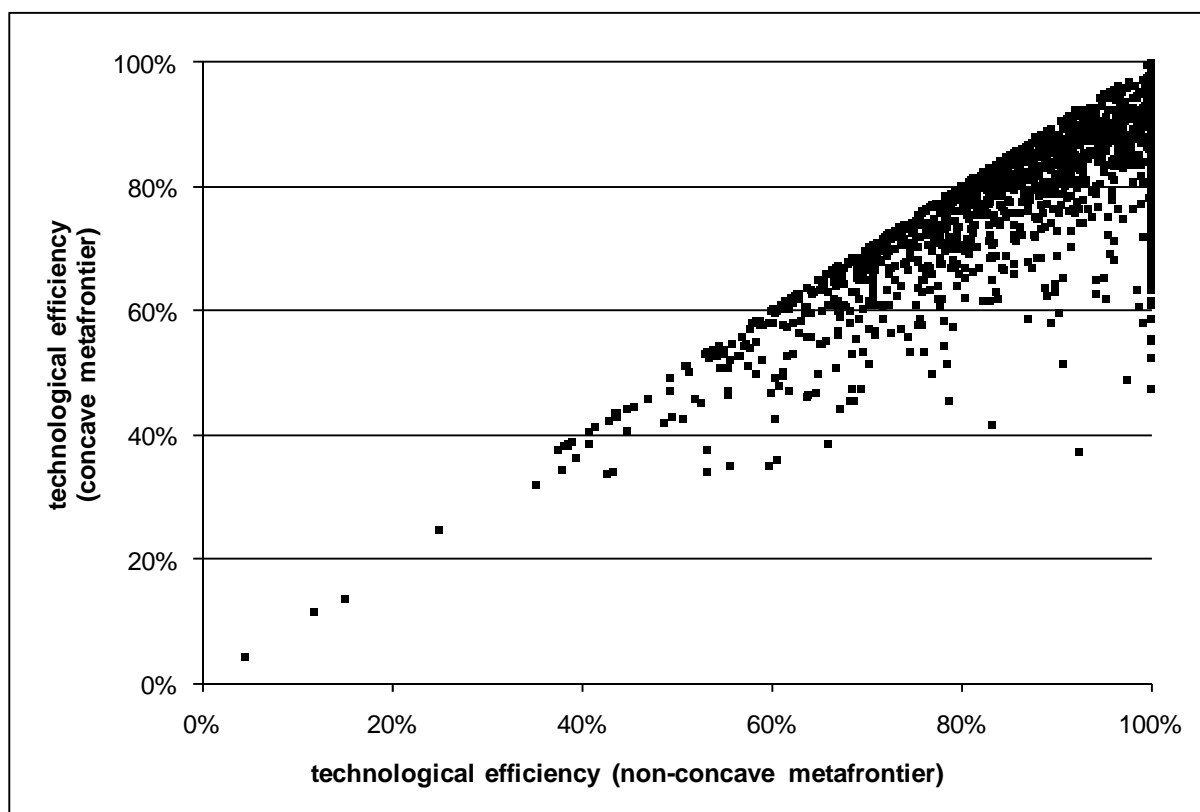
**Graph 4. Portion of technologically efficient farms**



Graph 3 shows that more than three fourth of the analysed farms in each specialisation class have a virtual metafrontier technology by means of the old literature approach. In specialisation class 4 even 85% of the farms have a virtual metafrontier technology. Taking into account the portion of technologically efficient farms from Graph 4 we can conclude that nearly every farm which is technologically inefficient by the old approach is compared to an input-output combination on the metafrontier that is not necessarily feasible for the farm technically. Consequently, on the one hand the portion of technologically efficient farms can be underestimated and on the other hand the potential output increase of the farms can be overestimated in the common literature approaches.

The former conclusion is also confirmed by Graph 4 for the analysis at hand. The portion of farms which are technologically efficient if we use the observed metafrontier is ten percentage points higher in specialisation class 1 and 55 percentage points higher in class 3. The differences of the remaining classes are between these extreme values. The latter conclusion is confirmed also by Graph 5. The technological efficiency evaluated by means of the concave metafrontier (values on the y-axis) is not larger for each farm and it is smaller for many farms than the potential output increase measured by the non-concave metafrontier (values on the x-axis). On average, the technological efficiency by means of the non-concave metafrontier is 6.7%-points higher for the inefficient farms. Table 3 shows that the choice of technology seems to be as important as technically efficient production because the related potentials for output increase are similar for inefficient farms. However, the average potential output increase is different among the specialisation classes.

**Graph 5. Technological efficiency by means of a concave and non-concave metafrontier**



**Table 3. Potential output increase of inefficient farms**

<b>potential output increase (%)</b>	<b>sc 1</b>	<b>sc 2</b>	<b>sc 3</b>	<b>sc 4</b>	<b>sc 5</b>
becoming technically efficient	51,1	61,8	51,2	52,4	32,6
becoming technologically efficient (non-concave metafrontier)	53,5	47,1	24,6	35,7	39,0
becoming technologically efficient (concave metafrontier)	65,6	59,2	18,9	41,9	50,2
Observations (all years)	110	291	1230	650	199

## 5 Conclusions

If group technologies cannot be combined such as organic and conventional farming as in Oude Lansink et al. (2002) the technological inefficiency is overestimated by the former literature approach for more than 75% of the observations in our sample. For our sample, the technological efficiency by means of the concave metafrontier is 6.7%-points lower compared to our approach that does not rely on the assumption of combinable technologies.

For empirical applications, we have to decide whether different technologies are combinable or not. If technologies are combinable our approach underestimates the inefficiency compared to the literature. If technologies are definitely not combinable – such as organic and conventional farming – the literature approach of a concave metafrontier is not appropriate. If one does not want to decide whether the technologies are combinable or not running both analyses seem to be useful for three reasons. (1) The non-concave metafrontier gives the maximum inefficiency, i.e. the highest potential of improving the farm's productivity a farmer

may achieve by changing the technology. (2) A farm that is inefficient following the non-concave metafrontier means there seems to be feasible potential to improve productivity by changing to an observed technology. (3) The target points following the non-concave metafrontier are easier to understand because target points of the concave metafrontier can be combinations of farms that are hardly combinable. From the latter two arguments follow many useful applications of the non-concave metafrontier besides the comparison of different technologies. Different ways of production, e.g. due to heterogeneity in inputs such as farmer's age, can be compared although they are not different technologies. For example, farms can be grouped by farmer's age to separate and quantify age's effect on the inefficiency from remaining determinants of inefficiency for each farmer individually.

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